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Peristaltic Motion of a Particle–Fluid Suspension in a Planar Channel

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We analyze the mechanics of peristaltic pumping of a particle–fluid suspension in a channel. A perturbation series (to second order) in dimensionless wave number of an infinite harmonic travelling wave is used to obtain an explicit form for the velocities and a relation between the flow rate and the pressure gradient in terms of the Reynolds number, concentration of the particles, suspension parameters, and the occlusion. We discuss the effect of the concentration of the particles, the Reynolds number, and the wave number on the pressure rise, peristaltic pumping, augmented pumping, and backward pumping. We also discuss the phenomenon of trapping.

1. INTRODUCTION

Peristaltic pumping has been the object of scientific and engineering research during the past few decades. It occurs due to the action of a progressive wave which propagates along the length of a distensible tube containing liquid. The pumping of fluids through muscular tubes by means of peristaltic waves is an important biological mechanism.

Study of the mechanism of peristalsis from both the mechanical and physiological viewpoints has been the object of scientific research. Since the first investigation of Latham (1996), several theoretical and experimental attempts have been made to understand peristaltic action in different situations. A review of much of the early literature is presented by Jaffrin and

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Shapiro (1971). A summary of most of the experimental and theoretical investigations reported with details of the geometry, fluid, Reynolds number, wavelength parameter, wave amplitude parameter, and wave shape has is given by Srivastava and Srivastava (1984).

Fluid dynamics of a particulate suspension (the suspended matter may consist of solid particles, liquid droplets, gas bubbles, etc.) has from historic times been the object of scientific and engineering research. Theoretical study of this fluid system is very useful in understanding various engineering problems concerned with powder technology, rain erosion in guided missiles, sedimentation, atmospheric fallout, combustion, fluidization, electrostatic precipitation of dust, nuclear reactor cooling, acoustics batch settling, aerosol and paint spraying, luner ash flows; in medicine, where erythrocyte sedimentation has become a standard clinical test; and in oceanography as well as other fields. The particulate suspension theory of blood has become the object of scientific research (Hill and Bedford, 1981; Srivastava and Srivastava, 1983; Trowbridge, 1984; Oka, 1985). The flows of suspensions of particles in a fluid have been studied by Marble (1971), Drew (1979), Bedford and Drumheller (1983), and Soo (1984). Applications of the theory of particle–fluid mixtures to microcirculation and erythrocyte sedimentation include the work of Bungay and Brenner (1973), Hill and Bedford (1981), Srivastava and Srivastava (1983), Trowbridge (1984), and Oka (1985). Peristaltic transport of a particle–fluid suspension was studied by Hung and Brown (1976), Kaimal (1978), and Srivastava and Srivastava (1989, 1995).

Most of the analytical studies use perturbation series in a small parameter such as the amplitude ratio or the dimensionless wave number, but it appears that no rigorous attempt has been made to study the effects of Reynolds number, wave number, and concentration of the particles on the pressure rise, peristaltic pumping, augmented pumping, and backward pumping for a particle-fluid suspension. *The purpose of this paper is to study the peristaltic pumping of a particle–fluid suspension in a planar channel.*

A regular perturbation series is used to solve the present problem; variables are expanded in a power series of the wave number α, which is defined as the ratio of half-width of the channel to the wavelength of the peristaltic wave. Closed-form solutions up to order α^2 are presented. The pressure rise per wavelength is obtained as a function of the time-averaged flow rate.

2. FORMULATION OF THE PROBLEM

Consider the two-dimensional flow of a mixture of small, spherical, rigid particles in an incompressible Newtonian viscous fluid in an infinite channel having width $2b$. We assume an infinite wave train traveling with velocity c along the walls. We choose a rectangular coordinate system for the channel with X along the centerline in the direction of wave propagation and Y transverse to it.

The equations governing the conservation of mass and linear momentum for both the fluid and particle phase using a continuum approach are expressed as follows (Drew, 1979; Srivastava and Srivastava, 1989):

Fluid phase:

$$(1 - C)\rho_f \frac{d\overline{W_f}}{dt} = -(1 - C)\nabla P + (1 - C)\mu_s(C)\nabla^2 \overline{W_f} + CS(\overline{W_p} - \overline{W_f}) \quad (1)$$

$$\nabla \cdot \overline{W_f} = 0 \quad (2)$$

Particulate phase:

$$C\rho_p \frac{d\overline{W_p}}{dt} = -C\nabla P + CS(\overline{W_f} - \overline{W_p}) \quad (3)$$

$$\nabla \cdot \overline{W_p} = 0 \quad (4)$$

In equations (1)–(4), $\overline{W_f}$, $\overline{W_p}$ denote fluid phase and particulate phase velocity vectors, respectively, d/dt denotes the material time derivative (the overbar refers to a dimensional quantity), ρ_f, ρ_p are the actual densities of the materials constituting the fluid and particulate phase, respectively, $(1 - C)\rho_f$, $C\rho_p$ denote the fluid phase density and the particulate phase density, respectively, P denotes the pressure, C is the volume fraction density of the particles, $\mu_s(C)$ is the particle fluid mixture viscosity, and S is the drag coefficient of the interaction for the force exerted by one phase on the other.

The concentration of the particles is considered so small that the field interaction between particles may be neglected. We choose the volume fraction density to be constant. The expression for the drag coefficient for the present problem is selected as

$$S = \frac{9}{2}\frac{\mu_0}{a^2}\lambda'(C)$$

$$\lambda'(C) = \frac{4 + 3\,[8C - 3C^2]^{1/2} + 3C}{[2 - 3C]^2} \quad (5)$$

where μ_0 is the fluid viscosity, and a is the radius of the particles. Relation (5) represents the classical Stokes drag for small particle Reynolds number, modified to account for the finite particulate fractional volume through the function $\lambda'(C)$ obtained by Tam (1969). We use the empirical relation for the viscosity of the suspension suggested by Charm and Kurland (1974),

$$\mu_s(C) = \mu_0 \frac{1}{1 - qC}$$

$$q = 0.07 \exp\left[2.49C + \frac{1107}{T}\exp(-1.69C)\right] \tag{6}$$

where T is the absolute temperature (K). The viscosity of the suspension expressed by this formula is found to be reasonably accurate up to $C = 0.6$.

Let $(\overline{U}_f, \overline{V}_f)$, $(\overline{U}_p, \overline{V}_p)$ be the velocity components for the fluid and particulate phases in the $\overline{X}$ and $\overline{Y}$ directions, respectively.

The geometry of the wall surface is defined as

$$\overline{h} = b + a \sin\left[\frac{2\pi}{\lambda}(\overline{X} - ct)\right] \tag{7}$$

and the boundary conditions are

$$\frac{\partial \overline{U}_f}{\partial \overline{Y}} = \frac{\partial \overline{U}_p}{\partial \overline{Y}} = 0, \qquad \overline{V}_f = \overline{V}_p = 0 \qquad \text{at} \quad \overline{Y} = 0$$

$$\overline{U}_f = 0 \qquad \text{at} \quad \overline{Y} = \overline{h} \tag{8}$$

where a is the wave amplitude and λ is the wavelength. We also assume the wall to have only a transverse motion.

We shall carry out this investigation in a coordinate system moving with the wave speed, in which the boundary shape is stationary. The coordinates and velocities in the laboratory frame $(\overline{X}, \overline{Y})$ and the wave frame $(\overline{x}, \overline{y})$ are related by

$$\begin{aligned} \overline{x} &= \overline{X} - ct, & \overline{y} &= \overline{Y} \\ \overline{u}_f &= \overline{U}_f - c, & \overline{u}_p &= \overline{U}_p - c \\ \overline{v}_f &= \overline{V}_f, & \overline{v}_p &= \overline{V}_p \end{aligned} \tag{9}$$

where $(\overline{u}_f, \overline{v}_f)$, $(\overline{u}_p, \overline{v}_p)$ are the velocity components in the wave frame.

If we employ these transformations in the governing equations of motion (1)–(4) and then introduce the dimensionless variables

$$x = \frac{2\pi\overline{x}}{\lambda}, \quad y = \frac{\overline{y}}{b}, \quad u_f = \frac{\overline{u}_f}{c}, \quad u_p = \frac{\overline{u}_p}{c}, \quad v_f = \frac{\overline{v}_f}{c}, \quad v_p = \frac{\overline{v}_p}{c}$$

$$h = \frac{\overline{h}(\overline{x})}{b}, \qquad P = \frac{2\pi b^2}{\lambda c \mu_s}\overline{P}(\overline{x}) \tag{10}$$

we find that the continuity equations, after defining the dimensionless stream functions $\Psi_f(x, y)$ and $\Psi_p(x, y)$ by

$$u_p = \frac{\partial \Psi_f}{\partial y}, \qquad u_p = \frac{\partial \Psi_p}{\partial y}, \qquad v_f = -\alpha \frac{\partial \Psi_f}{\partial x}, \qquad v_p = -\alpha \frac{\partial \Psi_p}{\partial x} \tag{11}$$

are satisfied identically, and after eliminating the pressure terms, the equations of motion become

$$(1 - C)\alpha \cdot Re\,[\Psi_{fy}\tilde{\nabla}^2 \Psi_{fx} - \Psi_{fx}\tilde{\nabla}^2\Psi_{fy}] = \tilde{\nabla}^2\tilde{\nabla}^2\Psi_f + CM(\tilde{\nabla}^2\Psi_p - \tilde{\nabla}^2\Psi_f) \tag{12}$$

$$C\alpha \cdot Re\,[\Psi_{py}\tilde{\nabla}^2\Psi_{px} - \Psi_{px}\tilde{\nabla}^2\Psi_{py}] = CN(\tilde{\nabla}^2\Psi_f - \tilde{\nabla}^2\Psi_p) \tag{13}$$

where the dimensionless wave number α, the Reynolds number Re, and the suspension parameters M and N are defined by

$$Re = \frac{cb\rho_f}{(1 - C)\mu_s}$$

$$M = \frac{Sb^2}{(1 - C)\mu_s}, \qquad N = \frac{Sb^2\rho_f}{(1 - C)\mu_s\rho_p}$$

$$\alpha = \frac{2\pi b}{\lambda}$$

and

$$\tilde{\nabla}^2 = \alpha^2 \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \tag{14}$$

3. RATE OF VOLUME FLOW AND BOUNDARY CONDITIONS

The instantaneous volume flow rate in the fixed frame is given by (Ungarish, 1993)

$$Q_f = (1 - C) \int_0^{\bar{h}} \bar{U}_f(\bar{X}, \bar{Y}, t)\, d\bar{Y} \tag{15}$$

$$Q_p = C \int_0^{\bar{h}} \bar{U}_p(\bar{X}, \bar{Y}, t)\, d\bar{Y} \tag{16}$$

$$Q_m = (1 - C) \int_0^{\bar{h}} \bar{U}_f(\bar{X}, \bar{Y}, t)\, d\bar{Y} + C \int_0^{\bar{h}} \bar{U}_p(\bar{X}, \bar{Y}, t)\, d\bar{Y} \tag{17}$$

where Q_f, Q_p, and Q_m are the volume flow rate for the fluid phase, particulate phase, and the mixture, respectively; h is a function of X and t.

The instantaneous volume flow rate in the wave frame is given by

$$q_f = (1 - C)\int_0^{\bar{h}} \bar{u}_f(\bar{x}, \bar{y})\, d\bar{y} \tag{18}$$

$$q_p = C\int_0^{\bar{h}} \bar{u}_p(\bar{x}, \bar{y})\, d\bar{y} \tag{19}$$

$$q_m = (1 - C)\int_0^{\bar{h}} \bar{u}_f(\bar{x}, \bar{y})\, d\bar{y} + C\int_0^{\bar{h}} \bar{u}_p(\bar{x}, \bar{y})\, d\bar{y} \tag{20}$$

where $\bar{h}$ is a function of $\bar{x}$ alone.

We shall be interested only with the volume flow rate of the fluid in this study. On substituting (9) into (15) and making use of (18), we find that the two rates of volume are related through

$$Q_f = q_f + (1 - C)c\bar{h} \tag{21}$$

The time-mean flow over a period T at a fixed position $\bar{X}$ is defined as

$$\bar{Q}_f = \frac{1}{T}\int_0^T Q_f\, dt \tag{22}$$

Substituting (21) into (22), and integrating, we get

$$\bar{Q}_f = q_f + (1 - C)ac \tag{23}$$

On defining the dimensionless time-mean flows θ and F, respectively, in the fixed and wave frame as

$$\theta \equiv \frac{\bar{Q}_f}{(1 - C)ac}, \qquad F \equiv \frac{q_f}{(1 - C)ac} \tag{24}$$

we find that (23) may be written as

$$\theta = F + 1 \tag{25}$$

where

$$F = \int_0^h \frac{\partial \Psi_f}{\partial y}\, dy = \Psi_f(h) - \Psi_f(0) \tag{26}$$

We note that h represents the dimensionless form of the surface of the peristaltic wall:

$$h(x) = 1 + \phi \sin x \tag{27}$$

where $\phi \equiv b/a$ is the amplitude ratio or occlusion. If we choose the zero value of the streamline at $(y = 0)$, then

$$\Psi_f(h) = F \tag{28}$$

The boundary conditions for the dimensionless stream function in the wave frame are

$$\begin{aligned} &\Psi_f = \Psi_p = 0, \qquad \Psi_{fyy} = \Psi_{pyy} = 0 \qquad \text{at} \quad y = 0 \\ &\Psi_{fy} = -1, \qquad \Psi_f = F \qquad \text{at} \quad y = h \end{aligned} \tag{29}$$

4. PERTURBATION SOLUTION

In order to solve the present problem, we expand the flow quantities in a power series of the small parameter α as (Siddiqui and Schwarz, 1994)

$$\Psi_f = \Psi_{f0} + \alpha\Psi_{f1} + \alpha^2\Psi_{f2} + \cdots$$

$$\Psi_p = \Psi_{p0} + \alpha\Psi_{p1} + \alpha^2\Psi_{p2} + \cdots$$

$$F = F_0 + \alpha F_1 + \alpha^2 F_2 + \cdots$$

and

$$\frac{\partial P}{\partial x} = \frac{\partial P_0}{\partial x} + \alpha\frac{\partial P_1}{\partial x} + \alpha^2\frac{\partial P_2}{\partial x} + \cdots \tag{30}$$

On substituting (30) into (12), (13), and (29) and collecting terms of equal powers of α and then equating the coefficients of like powers on both sides of the equations, we obtain the following set of problems.

System of Order Zero:

$$\Psi_{f0yyyy} + CM(\Psi_{p0yy} - \Psi_{f0yy}) = 0 \tag{31}$$

$$CN(\Psi_{f0yy} - \Psi_{p0yy}) = 0 \tag{32}$$

with the boundary conditions

$$\begin{aligned} &\Psi_{f0} = \Psi_{p0} = 0, \qquad \Psi_{f0yy} = \Psi_{p0yy} = 0 \qquad \text{at} \quad y = 0 \\ &\Psi_{f0} = F_0, \qquad \Psi_{f0y} = -1 \qquad \text{at} \quad y = h \end{aligned} \tag{33}$$

The solutions of the stream functions and the axial velocities are given as

$$\Psi_{f0} = -\frac{3}{2}(F_0 + h)\left(\frac{y^3}{3h^3} - \frac{y}{h}\right) - y \tag{34}$$

$$\Psi_{p0} = -\frac{3}{2}(F_0 + h)\left(\frac{y^3}{3h^3} - \frac{y}{h}\right) - y + \frac{3(F_0 + h)}{Mh^3}y \tag{35}$$

$$u_{f0} = -\frac{3}{2h}(F_0 + h)\left(\frac{y^2}{h^2} - 1\right) - 1 \tag{36}$$

$$u_{p0} = -\frac{3}{2h}(F_0 + h)\left(\frac{y^2}{h^2} - 1\right) - 1 + \frac{3(F_0 + h)}{Mh^3} \tag{37}$$

System of Order One:

$$(1 - C)\cdot Re[\Psi_{f0y}\Psi_{f0xyy} - \Psi_{f0x}\Psi_{f0yyy}] = \Psi_{f1yyyy} + CM(\Psi_{p1yy} - \Psi_{f1yy} \tag{38}$$

$$C\cdot Re[\Psi_{p0y}\Psi_{p0xyy} - \Psi_{p0x}\Psi_{p0yyy}] = CN(\Psi_{f1yy} - \Psi_{p1yy}) \tag{39}$$

The solutions of (38) and (39) subject to the first-order boundary conditions

$$\Psi_{f1} = \Psi_{p1} = 0, \qquad \Psi_{f1yy} = \Psi_{p1yy} = 0 \qquad \text{at} \quad y = 0 \tag{40}$$
$$\Psi_{f1} = F_1, \qquad \Psi_{f1y} = 0 \qquad \text{at} \quad y = h$$

are given as

$$\Psi_{f1} = \bar{b}_1(y^7 - 3h^4y^3 + 2h^6y) + \bar{b}_2(y^5 - 2h^2y^3 + h^4y) + \frac{3}{2}F_1\left(\frac{y}{h} - \frac{y^3}{3h^3}\right) \tag{41}$$

$$\begin{aligned}\Psi_{p1} = {} & \bar{b}_1\left(y^7 - 3h^4y^3 + 2h^6y - \frac{42}{M}y^5 + \frac{18}{M}h^4y\right) \\ & + \bar{b}_2\left(y^5 - 2h^2y^3 + h^4y - \frac{20}{M}y^3 + \frac{12}{M}h^2y\right) + (1 - C) \\ & \cdot Re\left(\frac{1}{M} - \frac{1}{N}\right)\left(\frac{1}{20}b_{10}y^5 + \frac{1}{6}b_{20}y^3\right) \\ & + E + \frac{3}{2}F_1\left(\frac{y}{h} + \frac{2y}{Mh^3} - \frac{y^3}{3h^3}\right)\end{aligned} \tag{42}$$

where

$$\bar{b}_1 = \frac{(1 - C)Re\bar{\beta}}{840}b_{10}, \qquad b_{10} = -\frac{3h'}{h^7}(3F_0^2 + 5F_0h + 2h^2)$$

$$\bar{b}_2 = \frac{(1 - C)Re\bar{\beta}}{120}b_{20}, \qquad b_{20} = \frac{3h'}{h^5}(3F_0^2 + 3F_0h + h^2)$$

$$E_1 = E_{10} + E_{11} + E_{12}, \qquad E_{10} = \frac{3(1-C)Re}{MN}\frac{(6F_0^2 + 6F_0 h + h^2)h'}{h^5}$$

$$E_{11} = -\frac{3}{4}(1-C)Re\left(\frac{1}{M}-\frac{1}{N}\right)\frac{F_0(3F_0+h)h'}{h^3},$$

$$E_{12} = -\frac{3(1-C)Re}{NM^2}\,b_{10}$$

$$\bar{\beta} = 1 + \frac{CM}{(1-C)N} \tag{43}$$

and the axial velocities at this order take the form

$$u_{f1} = \tilde{b}_1\,(7y^6 - 9h^4y^2 + 2h^6) + \tilde{b}_2\,(5y^4 - 6h^2y^2 + h^4) + \frac{3}{2}\frac{F_1}{h}\left(1-\frac{y^2}{h^2}\right) \tag{44}$$

$$\begin{aligned} u_{p1} = {} & \tilde{b}_1\left(7y^6 - 9h^4y^2 + 2h^6 - \frac{210}{M}y^4 + \frac{18}{M}h^4\right) \\ & + \tilde{b}_2\left(5y^4 - 6h^2y^2 + h^4 - \frac{60}{M}y^2 + \frac{12}{M}h^2\right) + (1-C) \\ & \cdot Re\left(\frac{1}{M}-\frac{1}{N}\right)\left(\frac{1}{4}b_{10}y^4 + \frac{1}{2}b_{20}y^2\right) \\ & + E_1 + \frac{3}{2}\frac{F_1}{h}\left(1 + \frac{2}{Mh^2} - \frac{y^2}{h^2}\right) \end{aligned} \tag{45}$$

System of Order Two:

$$\begin{aligned} & (1-C)\cdot Re[\Psi_{f1y}\Psi_{f0xyy} + \Psi_{f0y}\Psi_{f1xyy} - \Psi_{f1x}\Psi_{f0yyy} - \Psi_{f0x}\Psi_{f1yyy}] \\ & \quad = \Psi_{f2yyy} + 2\Psi_{f0xxyy} + CM(\Psi_{p2yy} - \Psi_{f2yy}) \end{aligned} \tag{46}$$

$$\begin{aligned} & C\cdot Re[\Psi_{p1y}\Psi_{p0xyy} + \Psi_{p0y}\Psi_{p1xyy} - \Psi_{p1x}\Psi_{p0yyy} - \Psi_{p0x}\Psi_{p1yyy}] \\ & \quad = CN(\Psi_{f2yy} - \Psi_{p2yy}) \end{aligned} \tag{47}$$

Using the zeroth-order and the first -order solutions in (46) and (47) and then applying the boundary conditions

$$\Psi_{f2} = \Psi_{p2} = 0, \qquad \Psi_{f2yy} = \Psi_{p2yy} = 0 \qquad \text{at} \quad y = 0 \tag{48}$$

$$\Psi_{f2} = F_2, \qquad \Psi_{f2y} = 0 \qquad \text{at} \quad y = h$$

we find that the stream functions Ψ_{f2}, Ψ_{p2} turn out to be

$$\Psi_{f2} = \frac{a_{21}}{660}\left(\frac{1}{12}y^{11} - \frac{5}{12}y^3h^8 + \frac{1}{3}yh^{10}\right) + \frac{a_{22}}{3024}(y^9 - 4y^3h^6 + 3yh^8)$$
$$+ \frac{a_{23}}{105}\left(\frac{1}{8}y^7 - \frac{3}{8}y^3h^4 + \frac{1}{4}yh^6\right) + \frac{a_{24}}{20}\left(\frac{1}{6}y^5 - \frac{1}{3}y^3h^2 + \frac{1}{6}yh^4\right)$$
$$+ \frac{3}{2}F_2\left(\frac{y}{h} - \frac{y^3}{3h^3}\right) \tag{49}$$

$$\Psi_{p2} = \frac{a_{21}}{660}\left[\frac{1}{12}y^{11} - \frac{5}{12}y^3h^8 + \frac{1}{3}yh^{10} - \frac{10}{12M}(11y^9 - 3yh^8)\right]$$
$$+ \frac{a_{22}}{3024}\left[y^9 - 4y^3h^6 + 3yh^8 - \frac{24}{M}(3y^7 - yh^6)\right]$$
$$+ \frac{a_{23}}{105}\left[\frac{1}{8}y^7 - \frac{3}{8}y^3h^4 + \frac{1}{4}yh^6 - \frac{3}{4M}(7y^5 - 3yh^4)\right]$$
$$+ \frac{a_{24}}{20}\left[\frac{1}{6}y^5 - \frac{1}{3}y^3h^2 + \frac{1}{6}yh^4 - \frac{2}{3M}(5y^3 - 3yh^2)\right]$$
$$+ z_1y^9 + z_2y^7 + z_3y^5 + z_4y^3 + z_5y$$
$$+ \frac{3}{2}F_2\left(\frac{y}{h} + \frac{2y}{Mh^3} - \frac{y^3}{3h^3}\right) \tag{50}$$

and the axial velocities at this order take the form

$$u_{f2} = \frac{a_{21}}{660}\left(\frac{11}{12}y^{10} - \frac{15}{12}y^2h^8 + \frac{1}{3}h^{10}\right) + \frac{a_{22}}{3024}(9y^8 - 12y^2h^6 + 3h^8)$$
$$+ \frac{a_{23}}{105}\left(\frac{7}{8}y^6 - \frac{9}{8}y^2h^4 + \frac{1}{4}h^6\right) + \frac{a_{24}}{20}\left(\frac{5}{6}y^4 - y^2h^2 + \frac{1}{6}h^4\right)$$
$$+ \frac{3}{2}F_2\left(\frac{1}{h} - \frac{y^2}{h^3}\right) \tag{51}$$

$$u_{p2} = \frac{a_{21}}{660}\left[\frac{11}{12}y^{10} - \frac{15}{12}y^2h^8 + \frac{1}{3}h^{10} - \frac{10}{12M}(99y^8 - 3h^8)\right]$$
$$+ \frac{a_{22}}{3024}\left[9y^8 - 12y^2h^6 + 3h^8 - \frac{24}{M}(21y^6 - h^6)\right]$$

$$+\frac{a_{23}}{105}\left[\frac{7}{8}y^6-\frac{9}{8}y^2h^4+\frac{1}{4}h^6-\frac{3}{4M}(35y^4-3h^4)\right]$$
$$+\frac{a_{24}}{20}\left[\frac{5}{6}y^4-y^2h^2+\frac{1}{6}h^4-\frac{2}{3M}(15y^2-3h^2)\right]$$
$$+9z_1y^8+7z_2y^6+5z_3y^4+3z_4y^2+z_5+\frac{3}{2}F_2\left(\frac{1}{h}+\frac{2}{Mh^3}-\frac{y^2}{h^3}\right) \quad (52)$$

where

$$a_{21}=(1-C)Re\overline{\beta}\,a_{10},$$

$$a_{22}=(1-C)\,Re\overline{\beta}a_{11}-(1-C)\,Re\cdot a_{12}+\frac{CRe}{N}a_{13}$$

$$a_{23}=(1-C)\,Re\cdot a_{14}+\frac{CRe}{N}a_{15},$$

$$a_{24}=(1-C)Re\cdot a_{16}+a_{17}+\frac{CRe}{N}a_{18}$$

$$a_{10}=-84\frac{(3F_0+2h)h'}{h^4}\tilde{b}_1-60\frac{(F_0+h)}{h^3}\tilde{b}_{1x},$$

$$a_{11}=315\frac{F_0h'}{h^2}\tilde{b}_1+21\frac{3F_0+h}{h}\tilde{b}_{1x}$$

$$a_{12}=15\frac{(3F_0+2h)h'}{h^4}\xi_1+27\frac{F_0+h}{h^3}\xi_{1x},$$

$$a_{13}=630\frac{(3F_0+2h)h'}{h^4}\tilde{b}_1+126\frac{F_0+h}{h^3}\tilde{b}_{1x}$$

$$a_{14}=90\frac{F_0h'}{h^2}\xi_1+10\frac{3F_0+h}{h}\xi_{1x}+6\frac{(3F_0+2h)h'}{h^4}\xi_2-6\frac{F_0+h}{h^3}\xi_{2x}$$

$$a_{15}=180\frac{(3F_0+2h)h'}{h^4}\xi_3+60\frac{F_0+h}{h^3}\xi_{3x}$$

$$a_{16}=9\frac{F_0h'}{h^2}\xi_2+3\frac{3F_0+h}{h}\xi_{2x}+3\frac{(3F_0+2h)h'}{h^4}\xi_4+3\frac{F_0+h}{h^3}\xi_{4x}$$

$$a_{17} = -6\,\frac{(3F_0 + 2h)h''}{h^4} + 36\,\frac{(2F_0 + h)h'^2}{h^5},$$

$$a_{18} = 18\,\frac{(3F_0 + 2h)h'}{h^4}\,\xi_5 + 18\,\frac{F_0 + h}{h^3}\,\xi_{5x}$$

$$\xi_1 = \overline{\beta}\,\tilde{b}_2 - \frac{CMRe}{20(1 - C)N^2}\,b_{10},$$

$$\xi_2 = \overline{\beta}\,\tilde{c}_1 - \frac{CMRe}{6(1 - C)N^2}\,b_{20}, \qquad \xi_3 = \tilde{b}_2 - \frac{Re}{20N}\,b_{10}$$

$$\xi_4 = \overline{\beta}\,\tilde{c}_3 + \frac{CM}{(1 - C)N}\left[E_1 - \frac{6}{M}\,\tilde{c}_1\right], \qquad \xi_5 = \tilde{c}_1 - \frac{Re}{6N}\,b_{20}$$

$$\tilde{c}_1 = -\frac{F_1}{2h^3} - 3\tilde{b}_1 h^4 - 2\tilde{b}_2 h^2, \qquad \tilde{c}_3 = \frac{3F_1}{2h} + 2\tilde{b}_1 h^6 + \tilde{b}_2 h$$

$$z_1 = \frac{1}{72}(1 - C)Re\left(\frac{1}{M} - \frac{1}{N}\right)a_1$$

$$z_2 = \frac{(1 - C)Re}{7}\left[-\frac{5}{2}\,\frac{(3F_0 + 2h)h'}{h^4}\,\omega_1 - \frac{9}{2}\,\frac{F_0 + h}{h^3}\,\omega_{1x} + \frac{105}{2}\,\frac{F_0 h'}{h^2}\left(\frac{1}{M} - \frac{1}{N}\right)\tilde{b}_1 + \frac{7}{2}\,\frac{3F_0 + h}{h}\,\tilde{b}_{1x} - \frac{105(3F_0 + 2h)h'}{MNh^4}\,\tilde{b}_1 - \frac{21}{MNh^3}\,\tilde{b}_{1x}\right]$$

$$z_3 = \frac{1}{5}(1 - C)Re\left[\frac{3}{2}\,\frac{(3F_0 + 2h)h'}{h^4}\,\omega_2 - \frac{3}{2}\,\frac{F_0 + h}{h^3}\,\omega_{2x} + \frac{45}{2}\,\frac{F_0 h'}{h^2}\,\omega_1 + \frac{5}{2}\,\frac{3F_0 + h}{h}\,\omega_{1x} - 45\frac{(3F_0 + 2h)h'}{MNh^4}\,\xi_3 - 15\,\frac{F_0 + h}{MNh^3}\,\xi_{3x}\right]$$

$$z_4 = \frac{a_{17}}{12M} + \frac{1}{3}(1 - C)Re\left[\frac{3}{2}\,\frac{(3F_0 + 2h)h'}{h^4}\,\omega_3 + \frac{3}{2}\,\frac{F_0 + h}{h^3}\,\omega_{3x} + \frac{9}{2}\,\frac{F_0 h'}{h^2}\,\omega_2 + \frac{3}{2}\,\frac{3F_0 + h}{h}\,\omega_{2x} - 9\,\frac{(3F_0 + 2h)h'}{MNh^4}\,\xi_5 - 9\,\frac{F_0 + h}{MNh^3}\,\xi_{5x}\right]$$

$$z_5 = \omega_{4x} + (1 - C)Re[\omega_{5x} - \omega_{6x}], \qquad \omega_1 = \frac{\tilde{b}_2}{M} - \frac{\xi_3}{N},$$

$$\omega_2 = \frac{\tilde{c}_1}{M} - \frac{\xi_5}{N}$$

$$\omega_3 = \left(\frac{1}{M} - \frac{1}{N}\right)\tilde{c}_3 - \frac{1}{N}\left(E_1 - \frac{6}{M}\tilde{c}_1\right), \qquad \omega_4 = \frac{3F_0 h'}{2Mh^2},$$

$$\omega_5 = \frac{3F_0 + h}{2Mh}\tilde{c}_3$$

$$\omega_6 = \frac{1}{N}\left(\frac{3F_0 + h}{2h} + \frac{3(F_0 + h)}{Mh^3}\right)\left(\tilde{c}_3 + E_1 - \frac{6}{M}\tilde{c}_1\right) \tag{53}$$

where the prime and the subscript x denote the derivative with respect to x.

The expressions for the stream functions $\Psi_f(x, y)$ and $\Psi_p(x, y)$, up to second order, may be respectively written as

$$\begin{aligned}
\Psi_f = & -\frac{3}{2}(F_0 + h)\left(\frac{y^3}{3h^3} - \frac{y}{h}\right) - y \\
& + \alpha\Bigg[\tilde{b}_1(y^7 - 3h^4y^3 + 2h^6y) + \tilde{b}_2(y^5 - 2h^2y^3 + h^4y) \\
& + \frac{3}{2}F_1\left(\frac{y}{h} - \frac{y^3}{3h^3}\right)\Bigg] \\
& + \alpha^2\Bigg[\frac{a_{21}}{660}\left(\frac{1}{12}y^{11} - \frac{5}{12}y^3h^8 + \frac{1}{3}yh^{10}\right) \\
& + \frac{a_{22}}{3024}(y^9 - 4y^3h^6 + 3yh^8) \\
& + \frac{a_{23}}{105}\left(\frac{1}{8}y^7 - \frac{3}{8}y^3h^4 + \frac{1}{4}yh^6\right) + \frac{a_{24}}{20}\left(\frac{1}{6}y^5 - \frac{1}{3}y^3h^2 + \frac{1}{6}yh^4\right) \\
& + \frac{3}{2}F_2\left(\frac{y}{h} - \frac{y^3}{3h^3}\right)\Bigg]
\end{aligned} \tag{54}$$

$$\Psi_p = -\frac{3}{2}(F_0 + h)\left(\frac{y^3}{3h^3} - \frac{y}{h}\right) - y + \frac{3(F_0 + h)}{Mh^3}y$$

$$
\begin{aligned}
&+\alpha\left[\tilde{b}_1\left(y^7-3h^4y^3+2h^6y-\frac{42}{M}y^5+\frac{18}{M}h^4y\right)\right.\\
&+\tilde{b}_2\left(y^5-2h^2y^3+h^4y-\frac{20}{M}y^3+\frac{12}{M}h^2y\right)\\
&\times(1-C)Re\left(\frac{1}{M}-\frac{1}{N}\right)\left(\frac{1}{20}b_{10}y^5+\frac{1}{6}b_{20}y^3\right)\\
&\left.+E_1y+\frac{3}{2}F_1\left(\frac{y}{h}+\frac{2y}{Mh^3}-\frac{y^3}{3h^3}\right)\right]\\
&+\alpha^2\left[\frac{a_{21}}{660}\left(\frac{1}{12}y^{11}-\frac{5}{12}y^3h^8+\frac{1}{3}yh^{10}-\frac{10}{12M}(11y^9-3yh^8)\right)\right.\\
&+\frac{a_{22}}{3024}\left(y^9-4y^3h^6+3yh^8-\frac{24}{M}(3y^7-yh^6)\right)\\
&+\frac{a_{23}}{105}\left(\frac{1}{8}y^7-\frac{3}{8}y^3h^4+\frac{1}{4}yh^6-\frac{3}{4M}(7y^5-3yh^4)\right)\\
&+\frac{a_{24}}{20}\left(\frac{1}{6}y^5-\frac{1}{3}y^3h^2+\frac{1}{6}yh^4-\frac{2}{3M}(5y^3-3yh^2)\right)\\
&\left.+z_1y^9+z_2y^7+z_3y^5+z_4y^3+z_5y+\frac{3}{2}F_2\left(\frac{y}{h}+\frac{2y}{Mh^3}-\frac{y^3}{3h^3}\right)\right]
\end{aligned}
\tag{55}
$$

The results of our analysis can be expressed to second order of the flow rate by defining

$$F^{(2)}\equiv F_0+\alpha F_1+\alpha^2F_2 \tag{56}$$

Then substituting $F_0=F^{(2)}-\alpha F_1-\alpha^2F_2$ into (54) and (55) and neglecting the terms greater than $O(\alpha^2)$, we obtain the second-order expression for the stream function $\Psi^{(2)}_{f,p}$ in terms of the second-order flow rate $F^{(2)}$:

$$
\begin{aligned}
\Psi^{(2)}_f=&-\frac{3}{2}(F^{(2)}+h)\left(\frac{y^3}{3h^3}-\frac{y}{h}\right)-y\\
&+\alpha[\tilde{B}_1(y^7-3h^4y^3+2h^6y)+\tilde{B}_2(y^5-2h^2y^3+h^4y)]\\
&+\alpha^2\left[\frac{A_{21}}{660}\left(\frac{1}{12}y^{11}-\frac{5}{12}y^3h^8+\frac{1}{3}yh^{10}\right)\right.
\end{aligned}
$$

$$+ \frac{A_{22}}{3024}(y^9 - 4y^3h^6 + 3yh^8)$$

$$+ \frac{A_{23}}{105}\left(\frac{1}{8}y^7 - \frac{3}{8}y^3h^4 + \frac{1}{4}yh^6\right)$$

$$+ \frac{A_{24}}{20}\left(\frac{1}{6}y^5 - \frac{1}{3}y^3h^2 + \frac{1}{6}yh^4\right)\Bigg] \tag{57}$$

$$\Psi_p^{(2)} = -\frac{3}{2}(F^{(2)} + h)\left(\frac{y^3}{3h^3} - \frac{y}{h}\right) - y + \frac{3(F^{(2)} + h)}{Mh^3}y$$

$$+ \alpha\Bigg[\tilde{B}_1\left(y^7 - 3h^4y^3 + 2h^6y - \frac{42}{M}y^5 + \frac{18}{M}h^4y\right)$$

$$+ \tilde{B}_2\Bigg(y^5 - 2h^2y^3 + h^4y - \frac{20}{M}y^3$$

$$+ \frac{12}{M}h^2y\Bigg) + (1 - C)Re\left(\frac{1}{M} - \frac{1}{N}\right)\left(\frac{1}{20}B_{10}y^5 + \frac{1}{6}B_{20}y^3\right) + e_1y\Bigg]$$

$$+ \alpha^2\Bigg[\frac{A_{21}}{660}\left(\frac{1}{12}y^{11} - \frac{5}{12}y^3h^8 + \frac{1}{3}yh^{10} - \frac{10}{12M}(11y^9 - 3yh^8)\right)$$

$$+ \frac{A_{22}}{3024}\left(y^9 - 4y^3h^6 + 3yh^8 - \frac{24}{M}(3y^7 - yh^6)\right)$$

$$+ \frac{A_{23}}{105}\left(\frac{1}{8}y^7 - \frac{3}{8}y^3h^4 + \frac{1}{4}yh^6 - \frac{3}{4M}(7y^5 - 3yh^4)\right)$$

$$+ \frac{A_{24}}{20}\left(\frac{1}{6}y^5 - \frac{1}{3}y^3h^2 + \frac{1}{6}yh^4 - \frac{2}{3M}(5y^3 - 3yh^2)\right)$$

$$+ Z_1y^9 + Z_2y^7 + Z_3y^5 + Z_4y^3 + Z_5y\Bigg] \tag{58}$$

and the axial velocities can be easily obtained, where

$$\tilde{B}_1 = \frac{(1 - C)Re\bar{\beta}}{840}B_{10}, \qquad B_{10} = -\frac{3h'}{h^7}(3F^{(2)2} + 5F^{(2)}h + 2h^2)$$

$$\tilde{B}_2 = \frac{(1 - C)Re\bar{\beta}}{120}B_{20}, \qquad B_{20} = \frac{3h'}{h^5}(3F^{(2)2} + 3F^{(2)}h + h^2)$$

$$e_{10} = \frac{3(1-C)Re}{MN}\frac{(6F^{(2)2} + 6F^{(2)}h + h^2)h'}{h^5}$$

$$A_{10} = -84\frac{(3F^{(2)} + 2h)h'}{h^4}\tilde{B}_1 - 60\frac{(F^{(2)} + h)}{h^3}\tilde{B}_{1x}$$

$$\tilde{k}_1 = -3\tilde{B}_1h^4 - 2\tilde{B}_2h^2, \qquad \tilde{k}_3 = 2\tilde{B}_1h^6 + \tilde{B}_2h^4$$

$$Z_2 = \frac{(1-C)Re}{7}\left[-\frac{5}{2}\frac{(3F^{(2)} + 2h)h'}{h^4}\Omega_1 - \frac{9}{2}\frac{F^{(2)} + h}{h^3}\Omega_{1x} + \frac{105}{2}\frac{F^{(2)}h'}{h^2}\left(\frac{1}{M} - \frac{1}{N}\right)\tilde{B}_1 + \frac{7}{2}\frac{3F^{(2)} + h}{h}\tilde{B}_{1x} - \frac{105(3F^{(2)} + 2h)h'}{MNh^4}\tilde{B}_1 - \frac{21}{MNh^3}\tilde{B}_{1x}\right]$$

$$\Omega_6 = \frac{1}{N}\left[\frac{3F^{(2)} + h}{2h} + \frac{3(F^{(2)} + h)}{Mh^3}\right]\left[\tilde{k}_3 + e_1 - \frac{6}{M}\tilde{k}_1\right] \tag{59}$$

and the other coefficients are the same as those previously defined.

5. PRESSURE GRADIENT

When the flow is steady in the wave frame, one can characterize the pumping performance by means of the pressure rise per wavelength. On substituting (30) into the dimensionless equations of motion and equating the coefficients of like powers of α on both sides of the equations. we obtain a set of partial differential equations for $\partial P_0/\partial x$, $\partial P_1/\partial x$, and $\partial P_2/\partial x$.

We define the dimensionless pressure rise per wavelength in the wave frame as

$$\Delta P_\lambda = \int_0^{2\pi}\frac{dP}{dx}\,dx \tag{60}$$

Since $\partial P/\partial x$ is periodic in x, the pressure rise per wavelength in the longitudinal direction is independent of y and the integral in (60) can be evaluated on the axis at $y = 0$ (Siddiqui and Schwarz, 1994). Putting (30) into (60), we obtain

$$\Delta P_\lambda = \Delta P_{\lambda 0} + \alpha\Delta P_{\lambda 1} + \alpha^2\Delta P_{\lambda 2} + \cdots \tag{61}$$

and we compute the pressure rise per wavelength at each order for a wall shape of the sinusoidal form defined by (27). Using the zeroth-, first-, and second-order terms for the pressure gradient in (60) and integrating from 0 to 2π, we obtain

$$\Delta P_{\lambda}^{(2)} = -3(1 - C)[F^{(2)} I_3^* + I_2^*] + \alpha^2 L \tag{62}$$

where

$$L = -(1 - C)\left[\frac{1}{264} S_{21} + \frac{1}{126} S_{22} + \frac{3}{140} S_{23} + \frac{1}{10} S_{24}\right]$$

$$S_{21} = -\frac{6}{35}(1 - C)^2\, Re^2 \bar{\beta}^2 [3F^{(2)3} I_3 + 17F^{(2)2} I_2 + 22F^{(2)}\, I_1 + 8I_0]$$

$$S_{22} = (1 - C) Re \bar{\beta} S_{11} - (1 - C) Re S_{12} + \frac{C Re}{N} S_{13},$$

$$S_{23} = (1 - C) Re S_{14} + \frac{C Re}{N} S_{15}$$

$$S_{24} = (1 - C) Re S_{16} + S_{17} + \frac{C Re}{N} S_{18}$$

and

$$S_{11} = \frac{126}{280}(1 - C) Re \bar{\beta}\, [3F^{(2)2} I_2 + 5F^{(2)}\, I_1 + 2I_0]$$

$$S_{12} = -\frac{3}{20}(1 - C) Re \bar{\beta}^2 [18F^{(2)3} I_3 + 57F^{(2)2} I_2 + 45F^{(2)} I_1 + 13I_0] - \frac{9}{10} \frac{CM Re}{(1 - C) N^2} [18F^{(2)3} I_5 + 69F^{(2)2} I_4 + 77F^{(2)} I_3 + 26I_2]$$

$$S_{13} = -\frac{27}{10}(1 - C) Re \bar{\beta}\, [6F^{(2)3} I_5 + 13F^{(2)2} I_4 + 9F^{(2)} I_3 + 2I_2]$$

$$S_{14} = -\frac{3}{35}(1 - C) Re \bar{\beta}^2 \left[33F^{(2)3} I_3 + 95F^{(2)2} I_2 + 70F^{(2)} I_1 + \frac{59}{3} I_0\right] - \frac{6CM Re}{(1 - C) N^2} [6F^{(2)3} I_5 + 15F^{(2)2} I_4 + 13F^{(2)} I_3 + 4I_2]$$

$$S_{15} = 6(1 - C) Re \bar{\beta} [6F^{(2)3} I_5 + 9F^{(2)2} I_4 + 5F^{(2)} I_3 + I_2] + \frac{36 Re}{N} [6F^{(2)3} I_7 + 13F^{(2)2} I_6 + 9F^{(2)} I_5 + 2I_4]$$

$$
\begin{aligned}
S_{16} &= \frac{3}{140}(1-C)Re\overline{\beta}^2[30F^{(2)3}I_3 + 70F^{(2)2}I_2 + 44F^{(2)}I_1 + 11I_0] \\
&\quad + \frac{9}{70}\frac{CRe\overline{\beta}}{N}[66F^{(2)3}I_5 + 87F^{(2)2}I_4 + 43F^{(2)}I_3 + 8I_2 \\
&\quad + \frac{18CRe}{N^2}[12F^{(2)3}I_7 + 18F^{(2)2}I_6 + 8F^{(2)}I_5 + I_4] \\
&\quad - \frac{9CMRe}{2N}\left[\frac{1}{M} - \frac{1}{N}\right][6F^{(2)3}I_5 + 5F^{(2)2}I_4 + F^{(2)}I_3] \\
&\quad + \frac{54CRe}{MN^2}[6F^{(2)3}I_9 + 13F^{(2)2}I_8 + 9F^{(2)}I_7 + 2I_6] \\
&\quad + \frac{3CMRe}{(1-C)N^2}[3F^{(2)2}I_4 + 3F^{(2)}I_3 + I_2] \\
S_{17} &= 12[3F^{(2)}I_3 + 2I_2] \\
S_{18} &= -\frac{9}{70}(1-C)\,Re\overline{\beta}\,[66F^{(2)3}I_5 + 87F^{(2)2}I_4 + 43F^{(2)}I_3 + 8I_2] \\
&\quad - \frac{18Re}{N}[6F^{(2)3}I_7 + 9F^{(2)2}I_6 + 5F^{(2)}I_5 + I_4]
\end{aligned} \tag{63}
$$

and we have

$$
\begin{aligned}
I_n &= \int_0^{2\pi} \frac{h'^2}{h^n}\,dx \\
&= \phi^2 \int_0^{2\pi} \frac{\cos^2 x}{(1+\phi\,\sin x)^n}, \\
I_0 &= \pi\phi^2, \\
I_1 &= 2\pi\,[1 - \sqrt{1-\phi^2}]
\end{aligned}
$$

and for $n > 1$

$$
I_n = \frac{1}{n-1}[I^*_{n-1} - I^*_{n-2}] \tag{64}
$$

where

$$I_n^* = \int_0^{2\pi} \frac{dx}{[1 + \phi \sin(x)]^n}, \qquad I_0^* = 2\pi,$$

$$I_1^* = \frac{2\pi}{(1 - \phi^2)^{1/2}}, \qquad I_2^* = \frac{2\pi}{(1 - \phi^2)^{3/2}}$$

$$I_3^* = \frac{\pi(2 + \phi^2)}{(1 - \phi^2)^{5/2}}, \qquad I_4^* = \frac{\pi(2 + 3\phi^2)}{(1 - \phi^2)^{7/2}}$$

and for $n > 4$

$$I_n^* = \frac{1}{1 - \phi^2}\left[\left(\frac{2n - 3}{n - 1}\right) I_{n-1}^* - \left(\frac{n - 2}{n - 1}\right) I_{n-2}^*\right] \tag{65}$$

Here $\Delta P_\lambda^{(2)}$ and $F^{(2)}$ are the pressure drop and the flow rate, respectively, in the wave frame to the second order in α,

$$\Delta P_\lambda^{(2)} = \Delta P_{\lambda 0} + \alpha \Delta P_{\lambda 1} + \alpha^2 \Delta P_\lambda, \qquad F^{(2)} = F_0 + \alpha F_1 + \alpha^2 F_2 \tag{66}$$

We also note that the relation between the dimensionless flow rate in the wave frame ($F^{(2)}$) and the time-mean flow rate in the fixed frame ($\theta^{(2)}$) is given by

$$\theta^{(2)} = F^{(2)} + 1 \tag{67}$$

6. NUMERICAL RESULTS AND DISCUSSION

We have obtained an analytical solution to the field equations for the peristaltic flow of an incompressible Newtonian fluid with suspended particles in a planar channel by using a regular perturbation series in terms of the dimensionless wave number α. The results of our analysis are presented as follows.

1. The pressure rise–flow rate relationship for the parameters Re, C, ϕ, and α.

2. The streamlines and trapping regions for the parameters Re, ϕ, α, C, and $\theta^{(2)}$.

6.1. Pressure Rise–Flow Rate Relations

Figure 1 is a graph of the dimensionless pressure change per wavelength $\Delta P_\lambda^{(2)}$ with the dimensionless flow rate ($\theta^{(2)}$) for the case $\{Re = 0, \phi = 0.3, C = 0.4, \alpha = 0, 1, 2, 3\}$. The graph is sectored so that the upper right-hand quadrant (I) denotes the region of peristaltic pumping. where $\theta^{(2)} > 0$ (positive

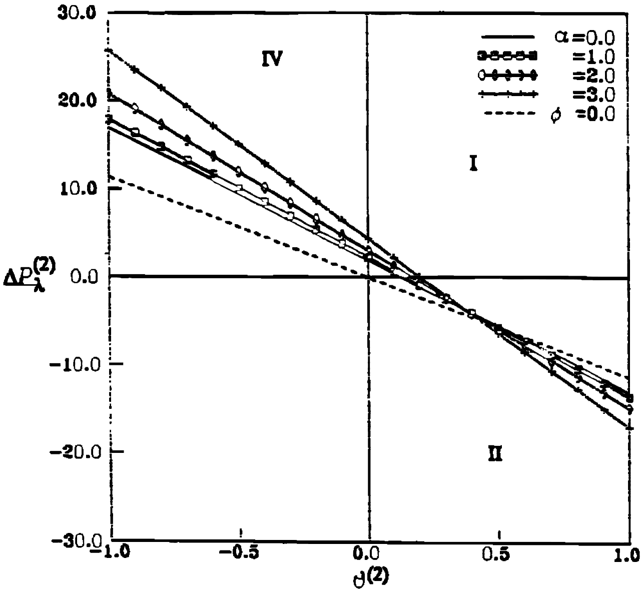

Fig. 1. Graph of the pressure gradient per wavelength $\Delta P_\lambda^{(2)}$ vs. dimensionless flow rate $\theta^{(2)}$ at $Re = 0.0$, $C = 0.4$, $\phi = 0.3$, and various values of the wave number α (---, $\phi = 0.0$, Poiseuille flow).

pumping) and $\Delta P_\lambda^{(2)} > 0$ (adverse pressure gradient). Quadrant II, where $\Delta P_\lambda^{(2)} < 0$ (favorable pressure gradient) and $\theta^{(2)} > 0$ (positive pumping), is designated as augmented flow. Quadrant IV, such that $\Delta P_\lambda^{(2)} > 0$ (adverse pressure gradient) and $\theta^{(2)} < 0$ is called retrograde or backward pumping, the flow is opposite to the direction of the peristaltic motion.

Figure 1 shows that the peristaltic pumping rate $\theta^{(2)}$ increases (for the same $\Delta P_\lambda^{(2)}$) as α increases for the case $\{Re = 0, \phi = 0.3, C = 0.4\}$. Also shown in Fig. 1 the case for $\phi = 0$, which is Poiseuille flow of a particle–fluid suspension between two plates.

Figure 2a is a graph of the pressure change per wavelength $\Delta P_\lambda^{(2)}$ vs. the observed flow rate $\theta^{(2)}$ for the case $\{Re = 10, \alpha = 0.06, \phi = 0.3, C = 0.0, 0.2, 0.4, 0.59\}$. Figure 2b is similar to Fig. 2a except that $\phi = 0.6$. We observe that an increase in C results in a decrease in the pumping rate if all other parameters are held fixed. Also, the backward pumping increases with increasing concentration of the particles.

Figure 3 shows the effect of the Reynolds number on the pumping rate for the case $\{C = 0.4, \alpha = 0.2, \phi = 0.8, Re = 0.0, 50, 100, 150\}$. We observe that an increase in Re results in an increase in the pumping rate if all other parameters are held fixed.

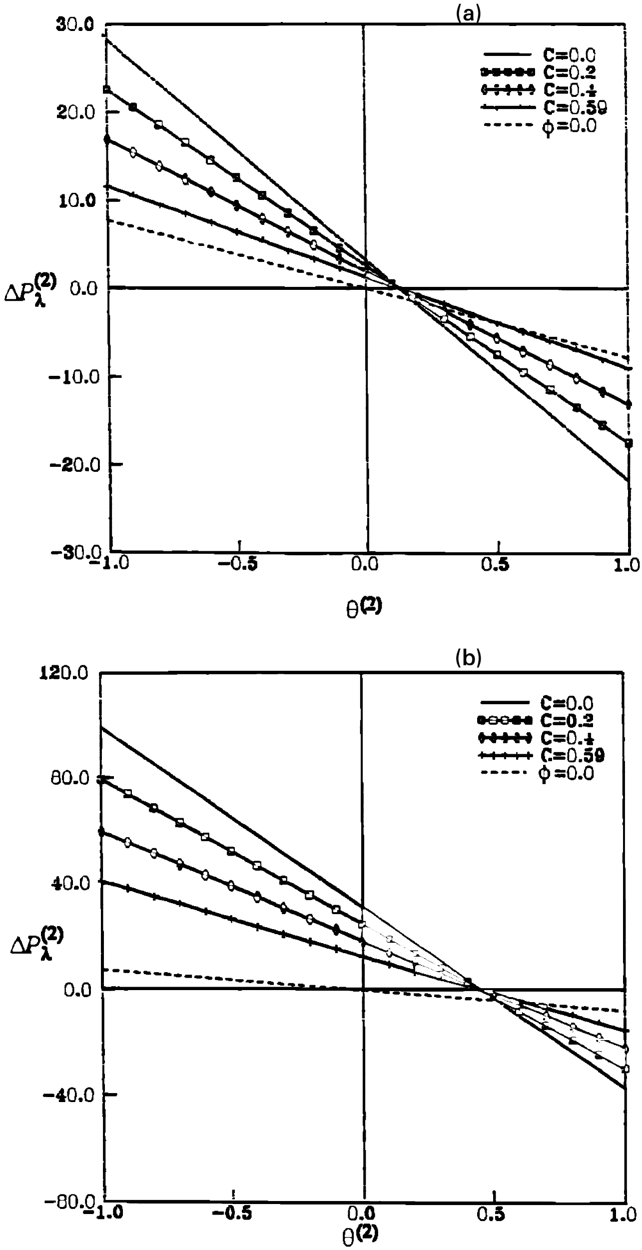

Fig. 2. Graph of the pressure gradient per wavelength $\Delta P_{\lambda}^{(2)}$ vs. dimensionless flow rate $\theta^{(2)}$ at $Re = 10$ and $\alpha = 0.06$, for various values of the concentration C, and (a) $\phi = 0.3$, or (b) $\phi = 0.6$ (---, $\phi = 0.0$, Poiseuille flow).

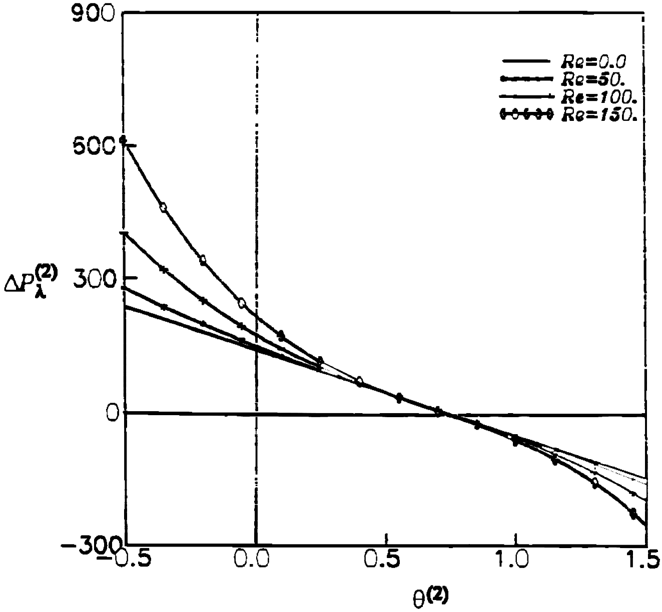

Fig. 3. Graph of the pressure gradient per wavelength $\Delta P_\lambda^{(2)}$ vs. dimensionless flow rate $\theta^{(2)}$ at $\alpha = 0.2$, $\phi = 0.8$, $C = 0.4$, and for various values of the Reynolds number *Re*.

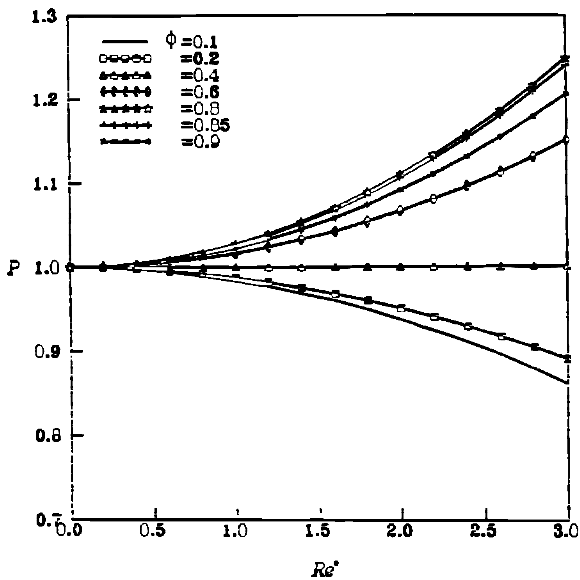

Fig. 4. Graph of the ratio of the pressure change per wavelength for zero peristaltic pumping to that for zero Reynolds number vs. *Re** at $C = 0.5$, $\alpha = 0.06$, and various values of the amplitude ratio ϕ.

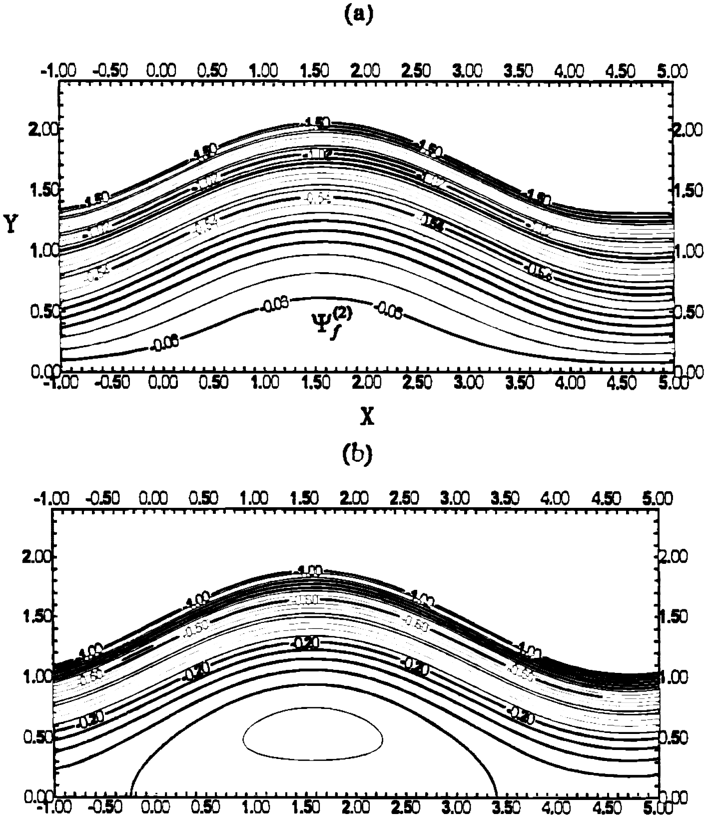

Fig. 5. Graph of the streamlines $\Psi_f^{(2)}$ at Re = 1., α = 0.0628, C = 0.0, ϕ = 0.4, and $\theta^{(2)}$ = (a) 0.5, (b) 0.7, (c) 1, (d) 2, (e) 5.

We define the pressure gradient required to obtain zero pumping ($\theta^{(2)}$ = 0) as $\Delta P^{(2)}_{\theta=0}$. Figure 4 is a graph of $P \equiv \Delta P^{(2)}_{\lambda}/\Delta P^{(2)}_{\theta=0}(Re^* = 0)$ vs. Re^* for varying occlusion ϕ at $C = 0.5$, $\alpha = 0.06$, which shows the effects of Re^* and ϕ on the pumping rate, where Re^* is the modified Reynolds number ($Re^* = Re\alpha/2\pi$) (Siddiqui and Schwarz 1994), which is the Stokes number.

6.2. Streamlines and Fluid Trapping

The phenomenon of trapping, whereby a bolus (defined as a volume of fluid bounded by closed streamlines in the wave frame) is transported at the wave speed, has been studied by several investigators (Shapiro *et al.*, 1969; Jaffrin, 1973; Siddiqui and Schwarz, 1993, 1994). Trapping occurs in a hyperspace of the variables ($\theta^{(2)}$, ϕ, Re, C, α). Here we have examined the case of small Reynolds number. Figures 5a–5e are graphs of streamlines for

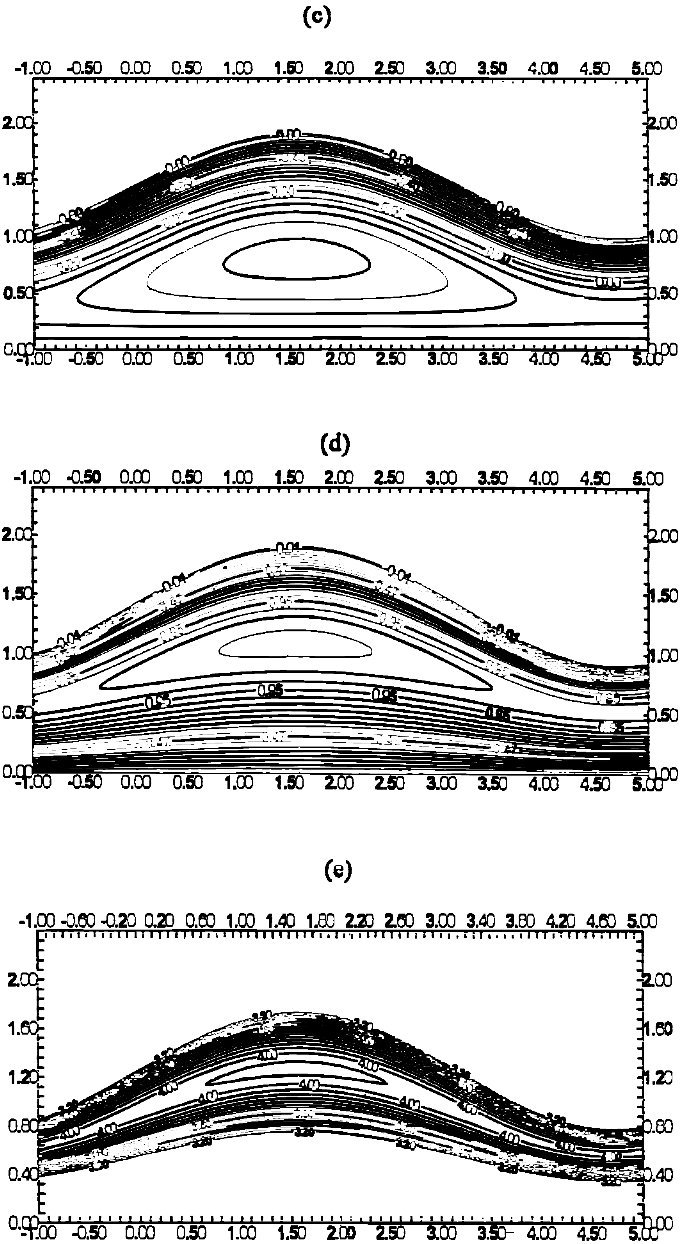

Fig. 5. Continued.

the conditions $\{Re = 1, \phi = 0.4, C = 0.0, \alpha = 0.0628$, and $\theta^{(2)} = 0.5, 0.7, 1, 2, 5\}$; Fig. 5a shows that there is no trapping region for peristaltic pumping; Fig. 5b shows the centerline trapped eddy, which is described by Siddiqui and Schwarz (1994); Figs. 5c–5e show that the trapped bolus is small as $\theta^{(2)}$ increases.

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